AdS/CFT - Playing the Devil's Advocate

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Well, there have always been those resistant to change

String Theory is No Exception!

I don't like that they're not calculating anything. I don't like that they don't check their ideas. I don't like that for anything that disagrees with an experiment, they cook up an explanation—a fix-up to say, "Well, it might be true." For example, the theory requires ten dimensions. Well, maybe there's a way of wrapping up six of the dimensions. Yes, that's all possible mathematically, but why not seven? When they write their equation, the equation should decide how many of these things get wrapped up, not the desire to agree with experiment. In other words, there's no reason whatsoever in superstring theory that it isn't eight out of the ten dimensions that get wrapped up and that the result is only two dimensions, which would be completely in disagreement with experience. So the fact that it might disagree with experience is very tenuous, it doesn't produce anything; it has to be excused most of the time. It doesn't look right.

– Richard Feynman (interview quoted in Davies & Brown, 1988)

The Large N Limit of Superconformal field theories and supergravity

Juan Maldacena

We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of Anti-deSitter spacetimes, spheres and other compact manifolds. This is shown by taking some branes in the full M/string theory and then taking a low energy limit where the field theory on the brane decouples from the bulk. We observe that, in this limit, we can still trust the near horizon geometry for large N. The enhanced supersymmetries of the near horizon geometry correspond to the extra supersymmetry generators present in the superconformal group (as opposed to just the super-Poincare group). The ‘t Hooft limit of 3+1 N = 4 super-Yang-Mills at the conformal point is shown to contain strings: they are IIB strings. We conjecture that compactifications of M/string theory on various Anti-deSitter spacetimes is dual to various conformal field theories. This leads to a new proposal for a definition of M-theory which could be extended to include five non-compact dimensions.

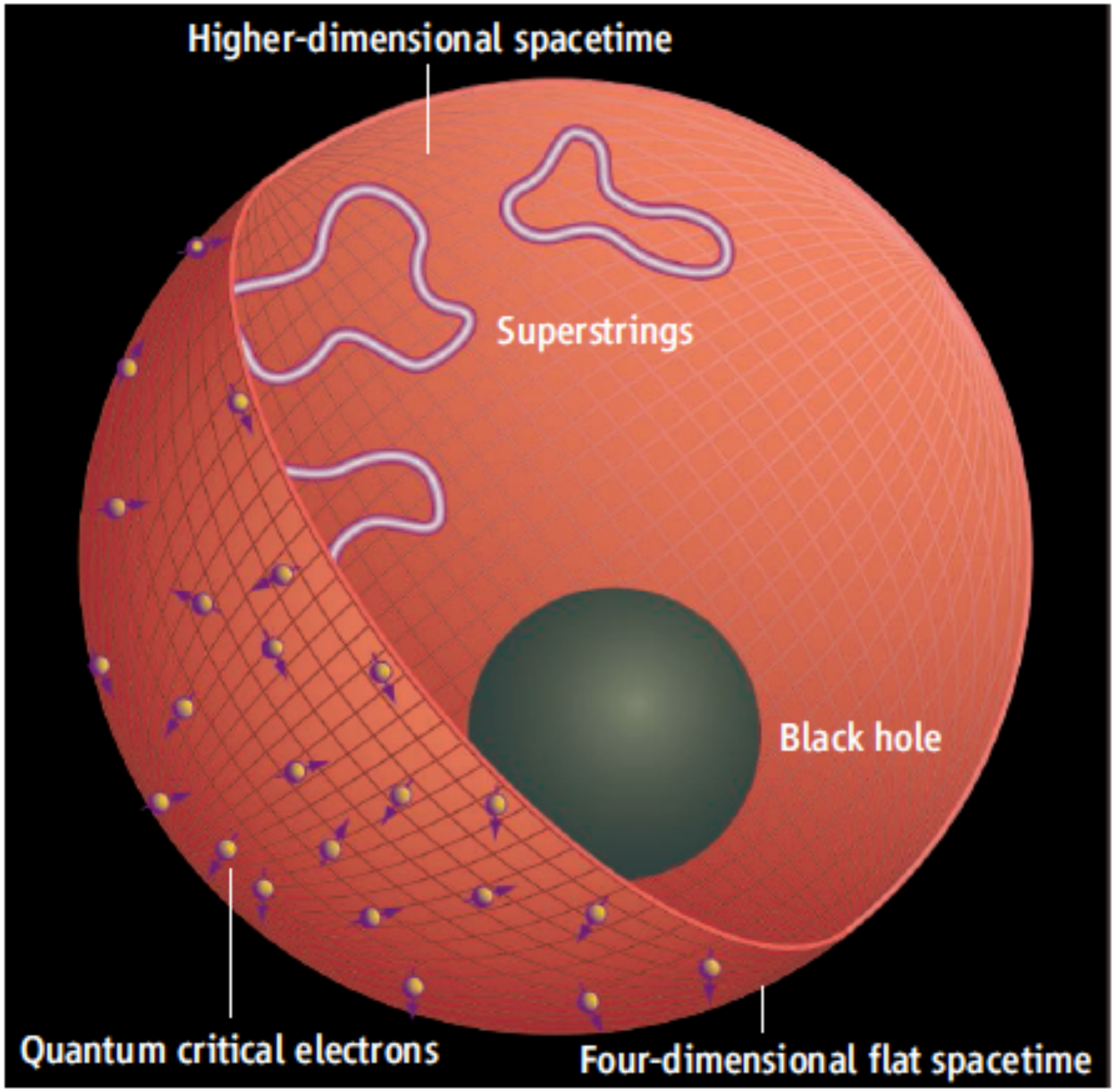
arXiv:hep-th/9711200

In simple English, the AdS/CFT conjecture is that certain strong coupling gauge theories of dimension d in their large N limit are dual to weakly coupled gravitational solutions in a d+1 dimensional Anti-deSitter spacetime.

The extra dimension in “d+1” is an RG scale that allows one to flow from the the boundary (i.e., the gauge theory in its UV limit) to near the event horizon of a black hole which sits at the center of this hyperbolic space. This near region defines the IR limit of the gauge theory.

Advantage - strong coupling non-perturbative method

Disadvantage - the conjecture has only been made for very special gauge theories, none of which seems to be related to Nature (i.e., QCD)



Hartnoll, Science (2008)

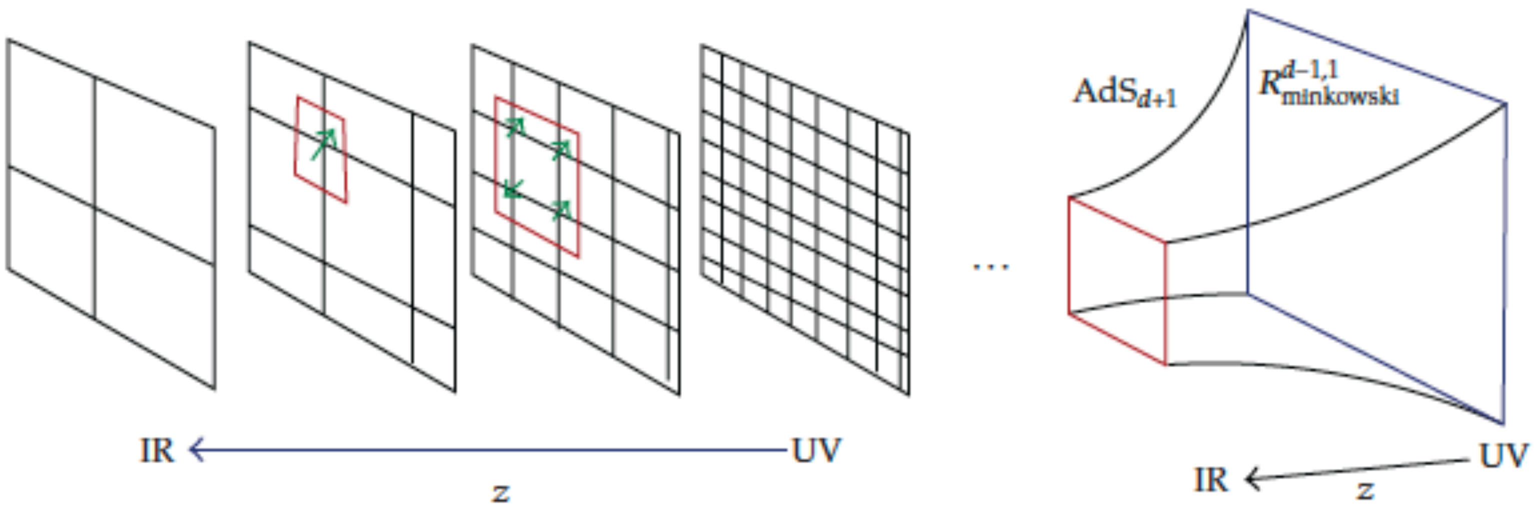


Figure 1: The extra (radial) dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image. The connection between these two pictures is pursued further in [15]. This paper contains a useful discussion of many features of the correspondence for those familiar with the real-space RG techniques developed recently from quantum information theory.

McGreevy, Adv. High Energy Phys. (2010)

Hence, their recent interest in Condensed Matter Physics

(i.e., we have LOTS of theories,
and LOTS of materials,
each of which has a different UV limit)

But, there are important "philosophical" differences …

It is natural to ask how surprised one should be that general relativity can reproduce the basic properties of superconductors. After all, Weinberg has shown that much of the phenomenology of superconductivity follows just from the spontaneous breaking of the U(1) symmetry. Once we have found the instability that leads to charged scalar hair, doesn't everything else follow?

Gary Horowitz - Introduction to Holographic Superconductors

arXiv:1002.1722

But their attention was focused on the details of the dynamics rather than the symmetry breaking. This is not just a matter of style. As BCS themselves made clear, their dynamical model was based on an approximation, that a pair of electrons interact only when the magnitude of their momenta is very close to a certain value, known as the Fermi surface. This leaves a question: How can you understand the exact properties of superconductors, like exactly zero resistance and exact flux quantization, on the basis of an approximate dynamical theory? It is only the argument from exact symmetry principles that can fully explain the remarkable exact properties of superconductors.

Steve Weinberg - From BCS to the LHC

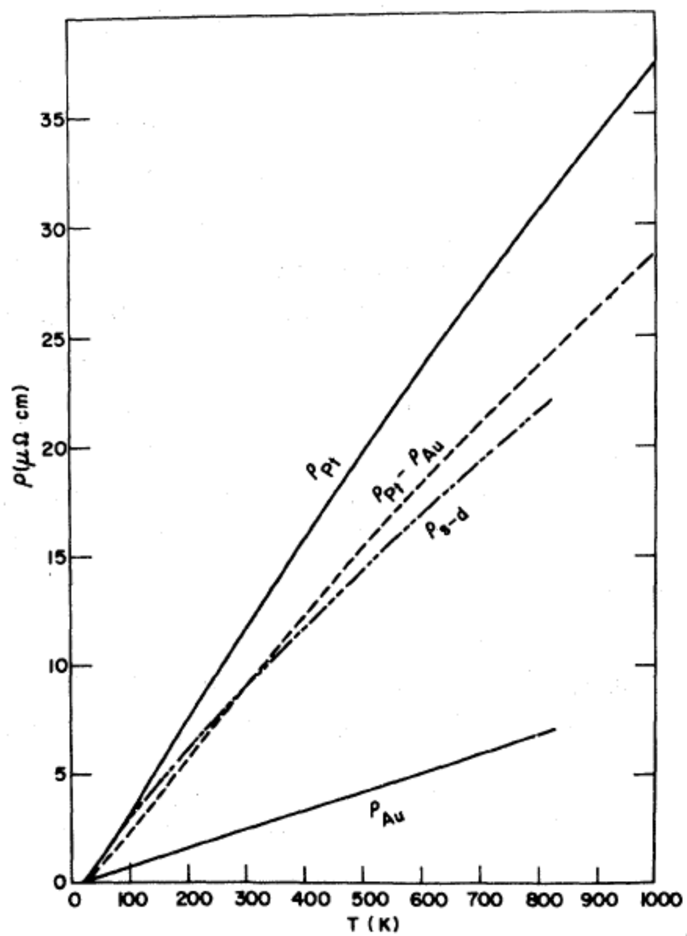
(see also Prog. Theor. Phys. Suppl. 86, 43 (1986))

In this study, the researchers focused on two properties that distinguish those cuprate strange metals from Fermi liquids. In ordinary Fermi liquids, electrical resistivity and the rates of electron scattering (deflection from their original course caused by interactions with each other) are both proportional to the temperature squared. However, in cuprates (and other superconducting non-Fermi liquids), electron scattering and resistivity are proportional to the temperature.

“There’s really no theory of how to explain that,” says Liu.

- MIT Press Release on Strange Metal Transport Realized by Gauge/Gravity Duality (Science 329, 1043 (2010))

Resistivity of Pt and Au



Fradin *et al*., PRB 12, 5570 (1975)

Holographic Superconductors

$$S = \int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\nabla\Psi - iqA\Psi|^2 - m^2|\Psi^2|\right)$$

where Ψ is a scalar field with charge q and mass m

(Note that the black hole defines a temperature T via the radius of its horizon, and a chemical potential, μ, through its charge)

$$m_{eff}^2 = m^2 + q^2 g^{tt} A_t^2$$

If $m_{eff}^2 < 0$, one has a superconducting solution (as in G-L theory)

Horowitz, arXiv:1002.1722

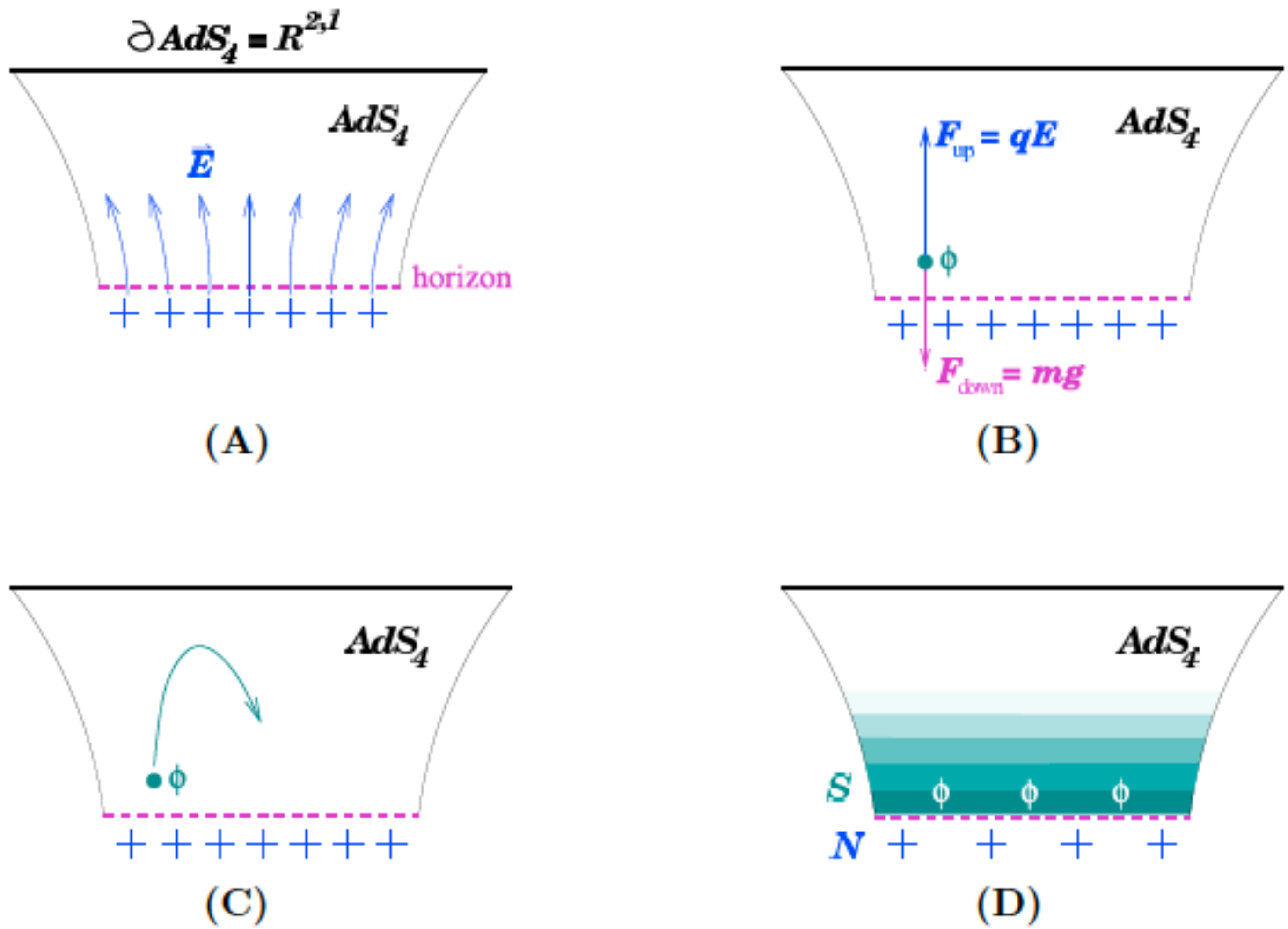
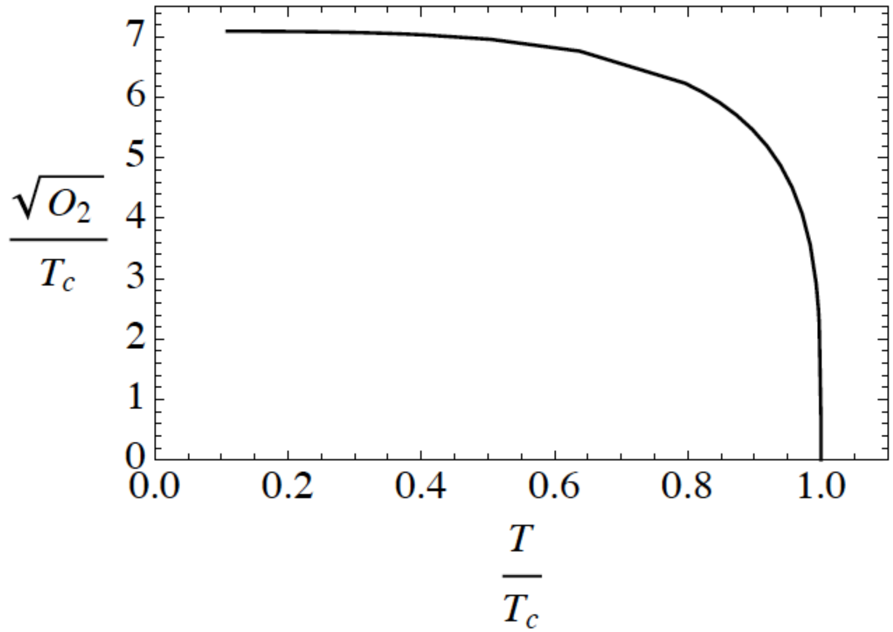
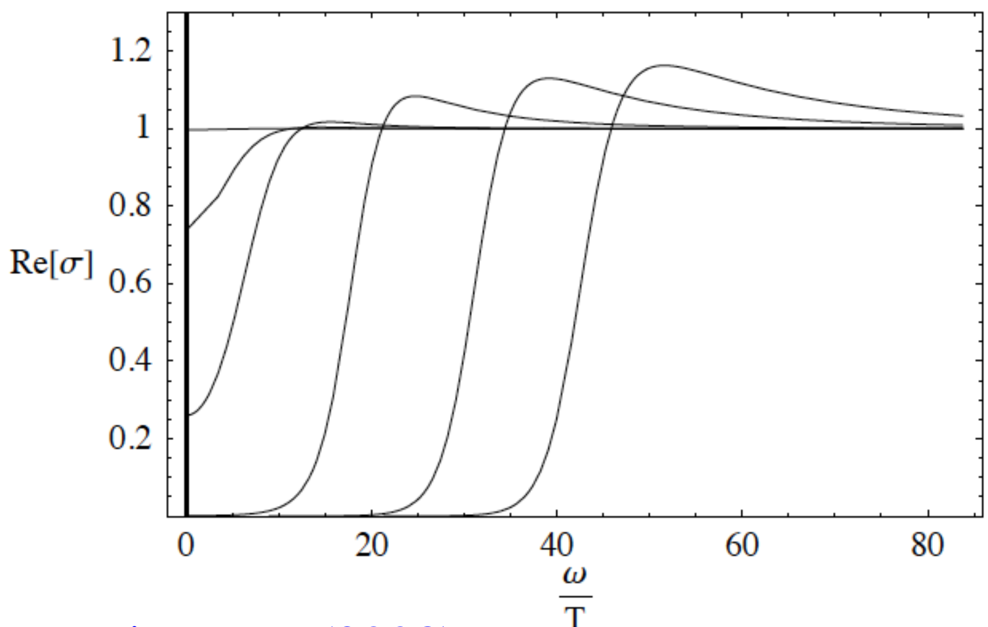


Fig. 4. A qualitative account of holographic superconductors, in pictures. (A) The normal state is described in terms of the RNAdS solution, with boundary $\mathbf{R}^{2,1}$. (B) The upward electrostatic force on a charged quantum of a scalar field ϕ can be greater than the downward gravitational pull. (C) The AdS_4 asymptotics prevent particles from escaping arbitrarily far from the horizon. (D) The quanta of ϕ instead condense just outside the horizon.

Gubser, arXiv:1012.5312





Hartnoll, Herzog, Horowitz, PRL (2008)

So, what's not to like?

There are no pairs! (the scalar field is put in by hand)

In BCS theory, <cc> → superconductivity

"Weinberg" philosophy, <Ψ> → superconductivity and <cc>

(as a consequence, the origin of the "gap" in σ is not clear)

So, what about d-wave?

Just use a spin 2 field instead!

But what about anisotropy ($d_{x^2-y^2}$)?

No big deal, just put in a field which breaks circular symmetry!

But what about the fermions?

Just put them in by hand too, but couple them to Ψ
the right way if you want to get something interesting …

Incorporating Fermions
(AdS-ARPES)

1. S. S. Lee - PRD 79, 086006 (2009)

2. Liu, McGreevy, Vegh –PRD 83, 065029 (2011)

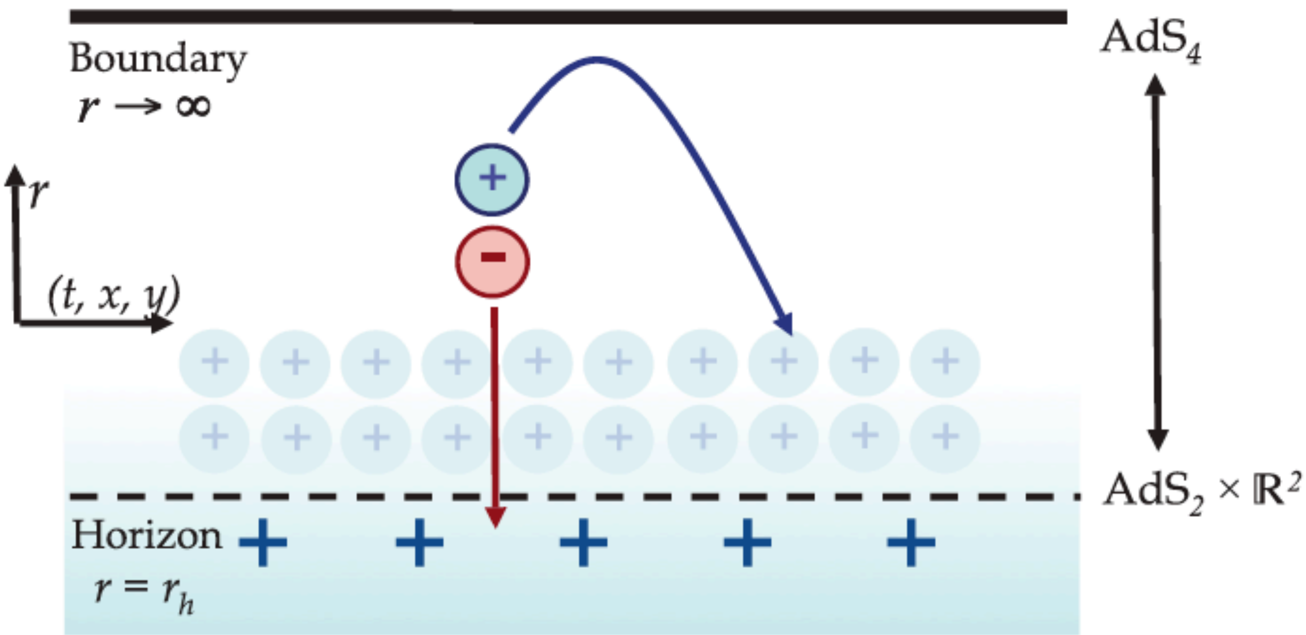
3. Cubrovic, Zaanen, Schalm - Science 325, 439 (2009)

4. Faulkner, Liu, McGreevy, Vegh - arXiv:0907.2694

5. Faulkner, Iqbal, Liu, McGreevy, Vegh - Science 329, 1043 (2010)

6. Chen, Kao, Wen - PRB 82, 026007 (2010)

7. Faulkner, Horowitz, McGreevy, Roberts, Vegh - JHEP 1003, 121 (2010)



$AdS_2 \rightarrow CFT_1$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[\mathcal{R} - \frac{6}{R^2} - \frac{R^2}{g_F^2} F_{MN} F^{MN} \right]$$

$$S_{\mathrm{spinor}} = \int d^{d+1}x \sqrt{-g}\, i(\bar{\psi}\Gamma^M \mathcal{D}_M \psi - m\bar{\psi}\psi)$$

Faulkner *et al*, Science (2010)

$$\delta_k = \frac{1}{2} + \nu_k,$$

$$\nu_k = \frac{1}{\sqrt{6}}\sqrt{m^2R^2 + \frac{3k^2}{\mu^2} - \frac{q^2}{2}},$$

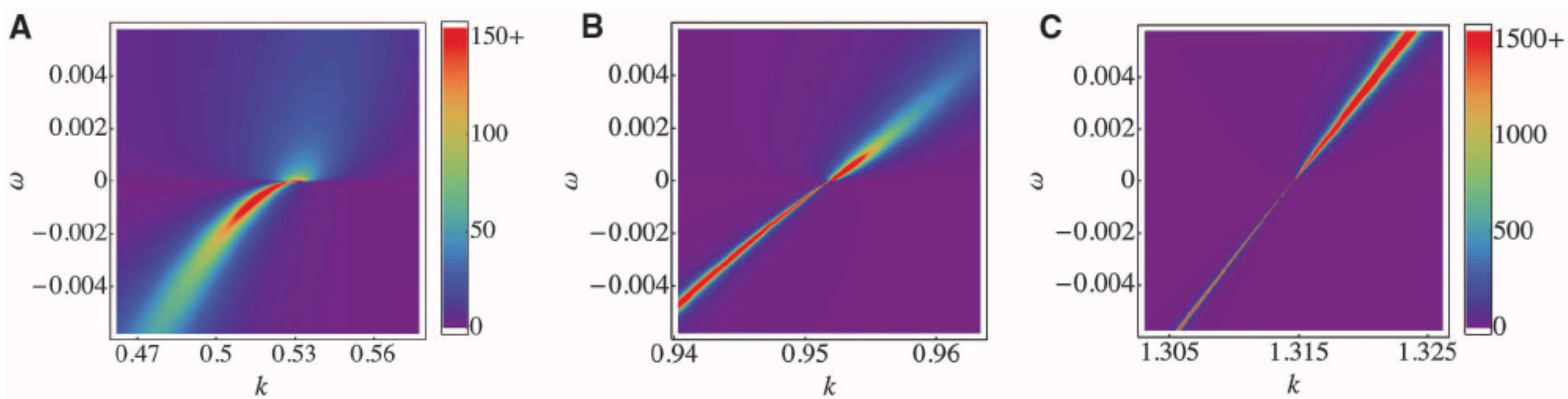
$$k = |\vec{k}|$$

$$G_R(k,\omega) = \frac{h_1}{k - k_F - \frac{1}{v_F}\omega - \Sigma(\omega,k)}$$

$$\Sigma(\omega,k) = h\mathcal{G}_{k_F}(\omega)$$
$$= hc(k_F)\omega^{2\nu_{k_F}}$$

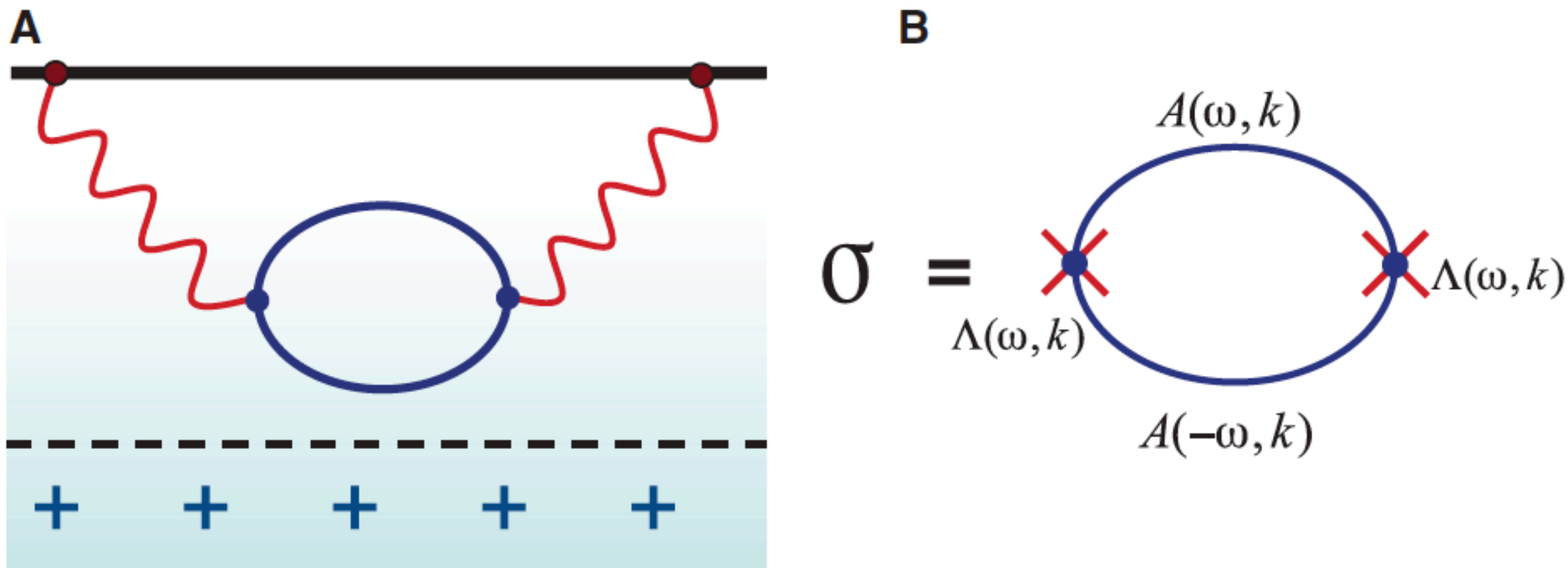
Faulkner *et al*, Science (2010)

There is a “Fermi surface” with gapless excitations
By tuning q, m, etc., one can get a FL, a non FL, or a marginal FL



Faulkner *et al*, Science (2010)

To get the conductivity, just calculate the "Kubo" bubble



$$\sigma_{DC} = \alpha(q,m)\, T^{-2\nu_{k_F}}$$

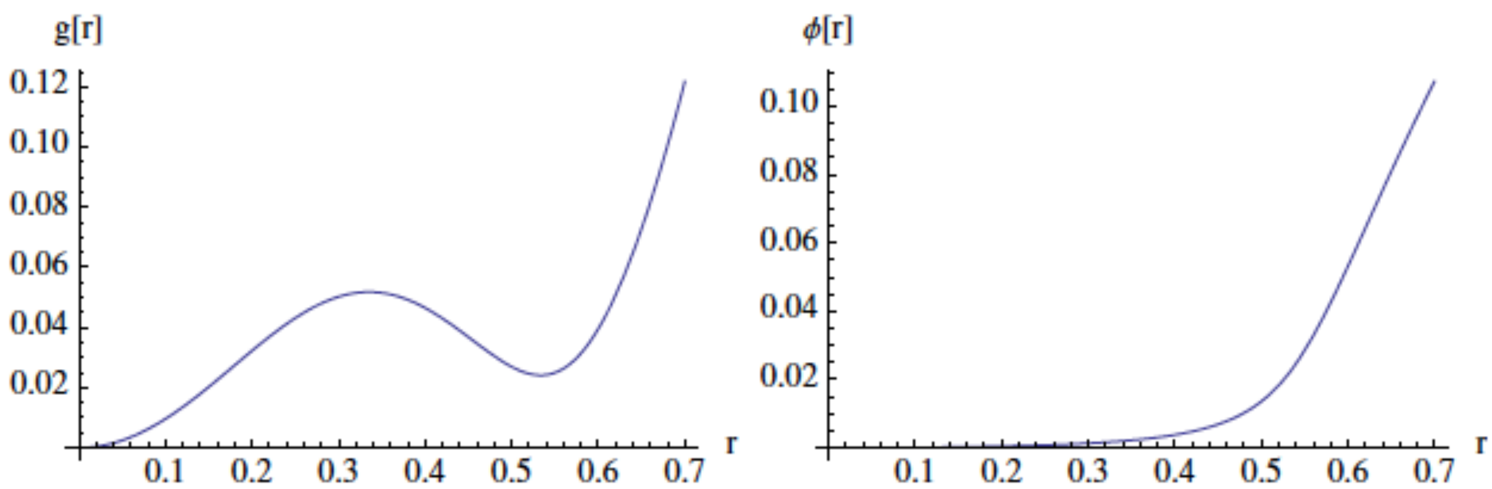
$$\sigma(\omega) \propto (i\omega)^{-2\nu_{k_F}}, \quad T \ll \omega \ll \mu$$

Faulkner *et al*, Science (2010)

Now add BOTH a scalar and a spinor field, and then couple the two

$$S[\zeta] = \int d^{d+1}x\sqrt{-g}\left[i\bar{\zeta}\left(\Gamma^M D_M - m_\zeta\right)\zeta + \eta_5^\star \varphi^\star \zeta^T C\Gamma^5\zeta + \eta_5\varphi\bar{\zeta}C\Gamma^5\bar{\zeta}^T\right]$$

(Γ^5 term mixes k and –k)

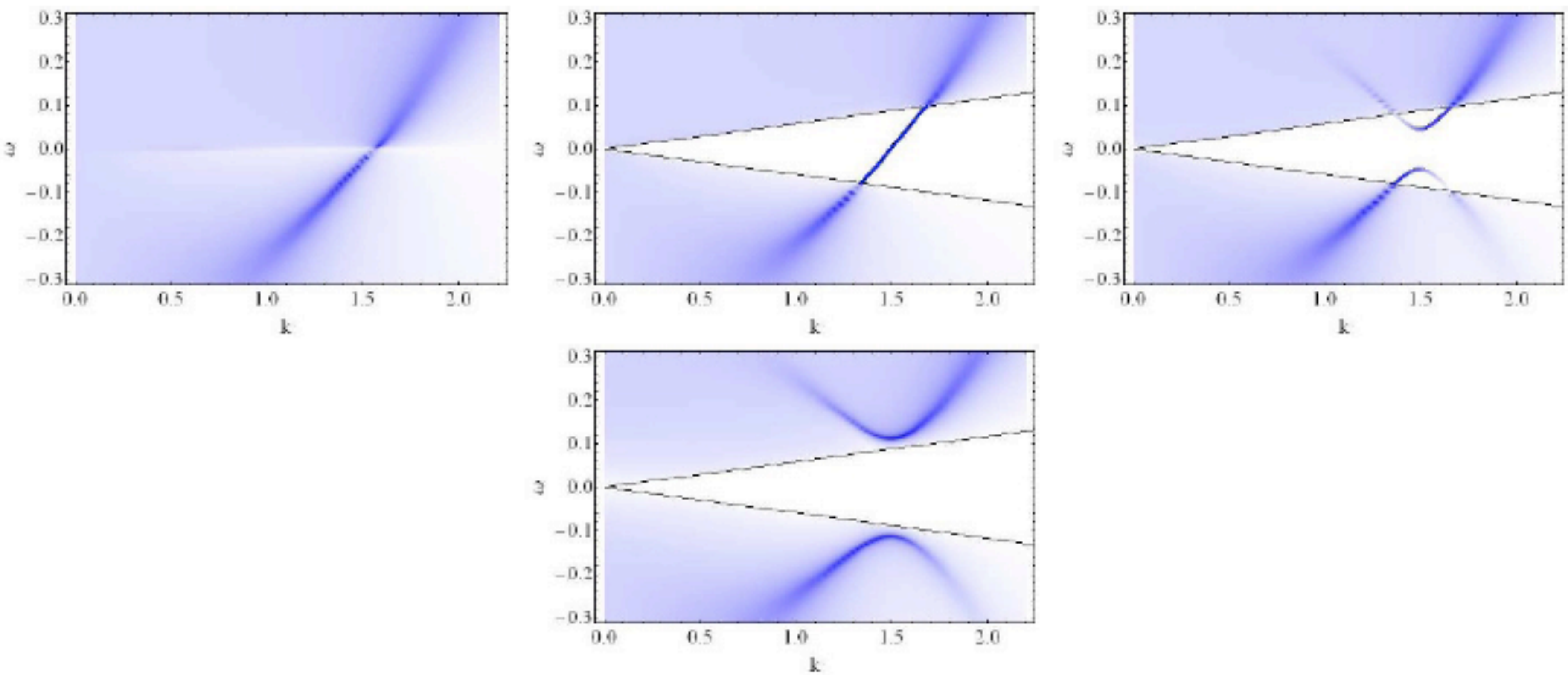


$AdS^2 \times R^2 \rightarrow AdS^4$

Faulkner *et al*, JHEP (2010)

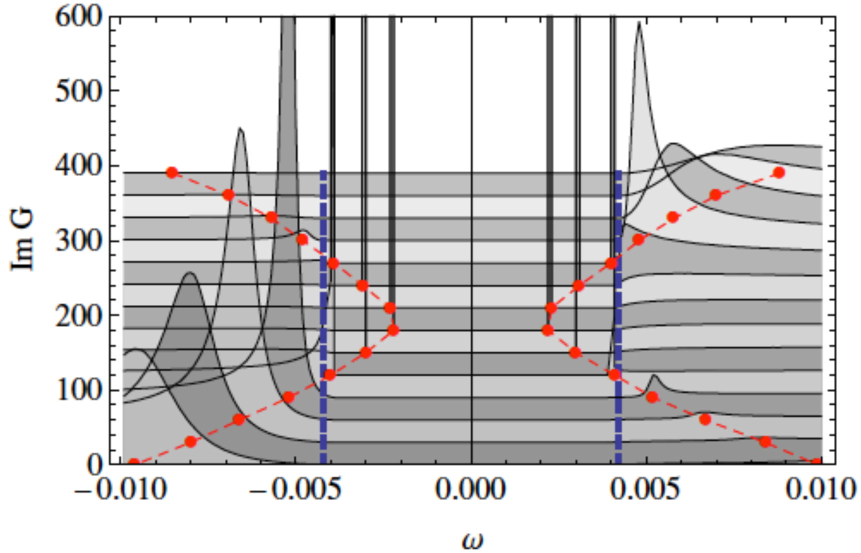
This change in geometry due to the scalar causes the "light cone" to open up ("timelike" bound states are quasiparticles)

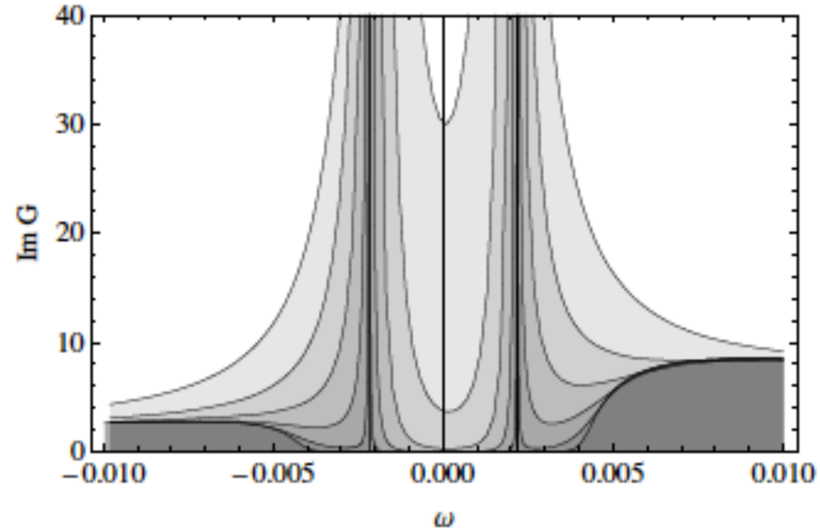
Coupling of the scalar to the spnior opens up a "Bogoliubov" gap

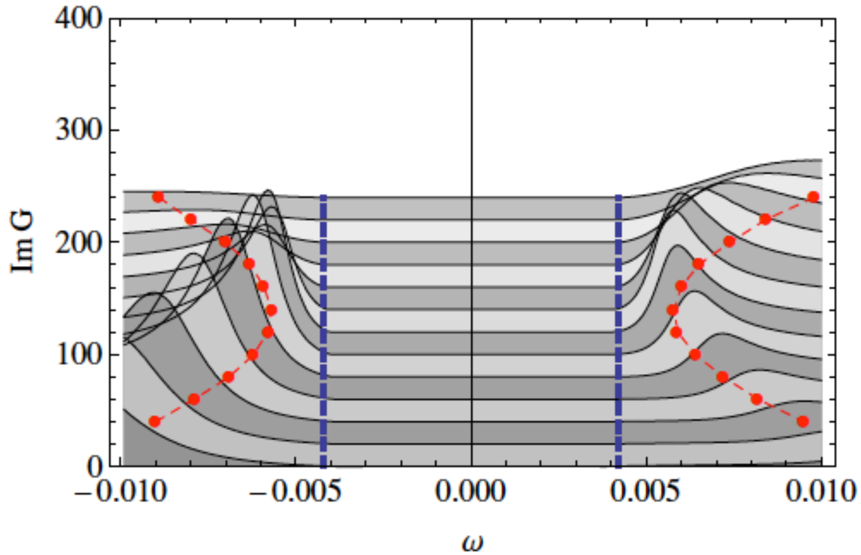


Faulkner *et al*, JHEP (2010)

And, voilà, out pops the peak/dip/hump





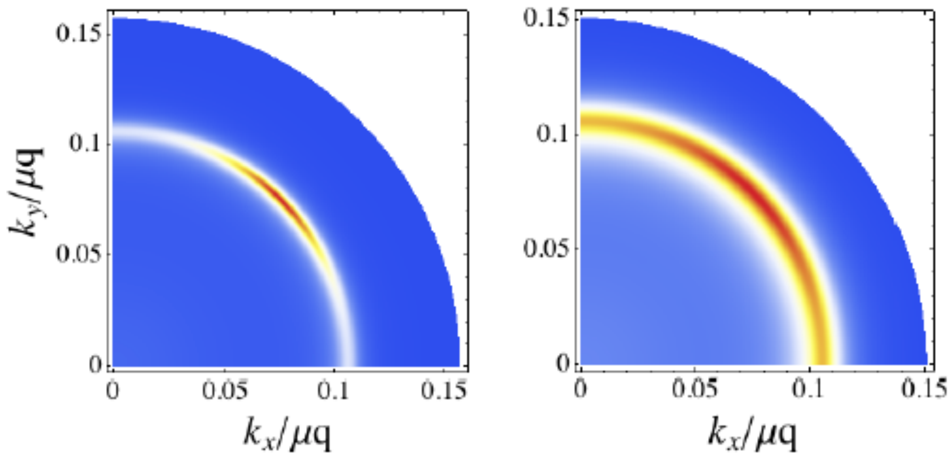


peak – bound state outside of the IR light cone, gapped due to the Γ^5 term (scalar-spinor coupling)

dip/hump – measures the "speed of light" of the quantum critical CFT

Faulkner *et al*, JHEP (2010)

And, you can get Fermi arcs too!



Benini, Herzog, Yaron, arXiv:1006.0731

$$\boxed{\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - iq_\varphi F_{\mu\nu}\varphi^{\mu\rho*}\varphi_\rho^\nu \\ &- |D_\rho\varphi_{\mu\nu}|^2 + 2|\varphi_\mu|^2 + |D_\mu\varphi|^2 - [\varphi^{\mu*}D_\mu\varphi + \mathrm{h.c.}] \\ &- m_\varphi^2\left(|\varphi_{\mu\nu}|^2 - |\varphi|^2\right) + 2R_{\mu\rho\nu\lambda}\varphi^{\mu\nu*}\varphi^{\rho\lambda} - \frac{R}{4}|\varphi|^2\end{aligned}}$$

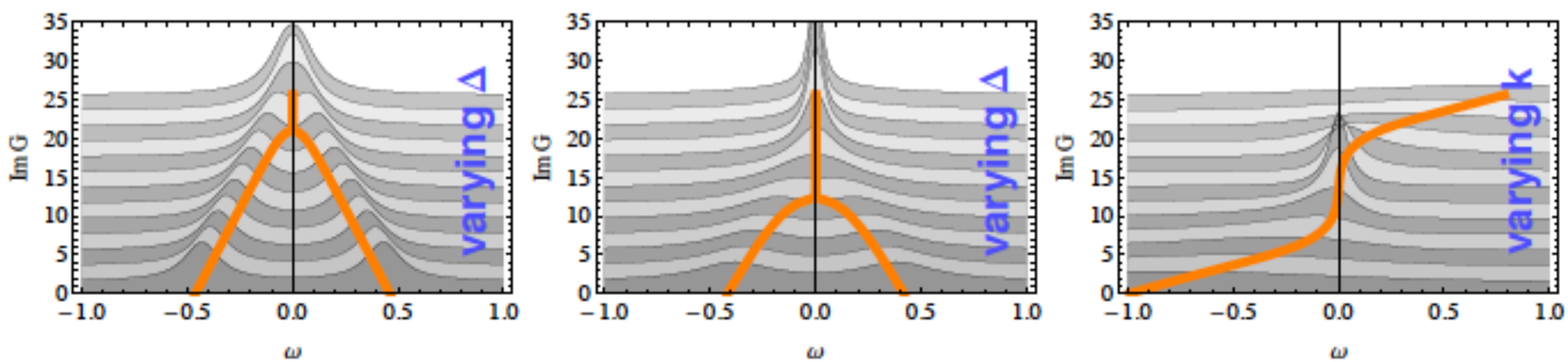
$$\boxed{A_t(z)\,, \qquad \varphi_{xy}(z)\,, \qquad \varphi_{xx} = -\varphi_{yy} \equiv \varphi_\Delta(z)}$$

$$\boxed{\begin{aligned}\mathcal{L}_\Psi = &\, i\overline{\Psi}\left(\Gamma^\mu D_\mu - m\right)\Psi \\ &+ \eta^*\varphi_{\mu\nu}^*\,\overline{\Psi^c}\Gamma^\mu D^\nu\Psi - \eta\,\overline{\Psi}\Gamma^\mu D^\nu(\varphi_{\mu\nu}\Psi^c)\end{aligned}}$$

Benini, Herzog, Yaron, arXiv:1006.0731

$$\boxed{b(r) = g_1 \int_{r_*}^{r} dr' \sqrt{g_{rr}} h(r') B_{xy}(r')}$$

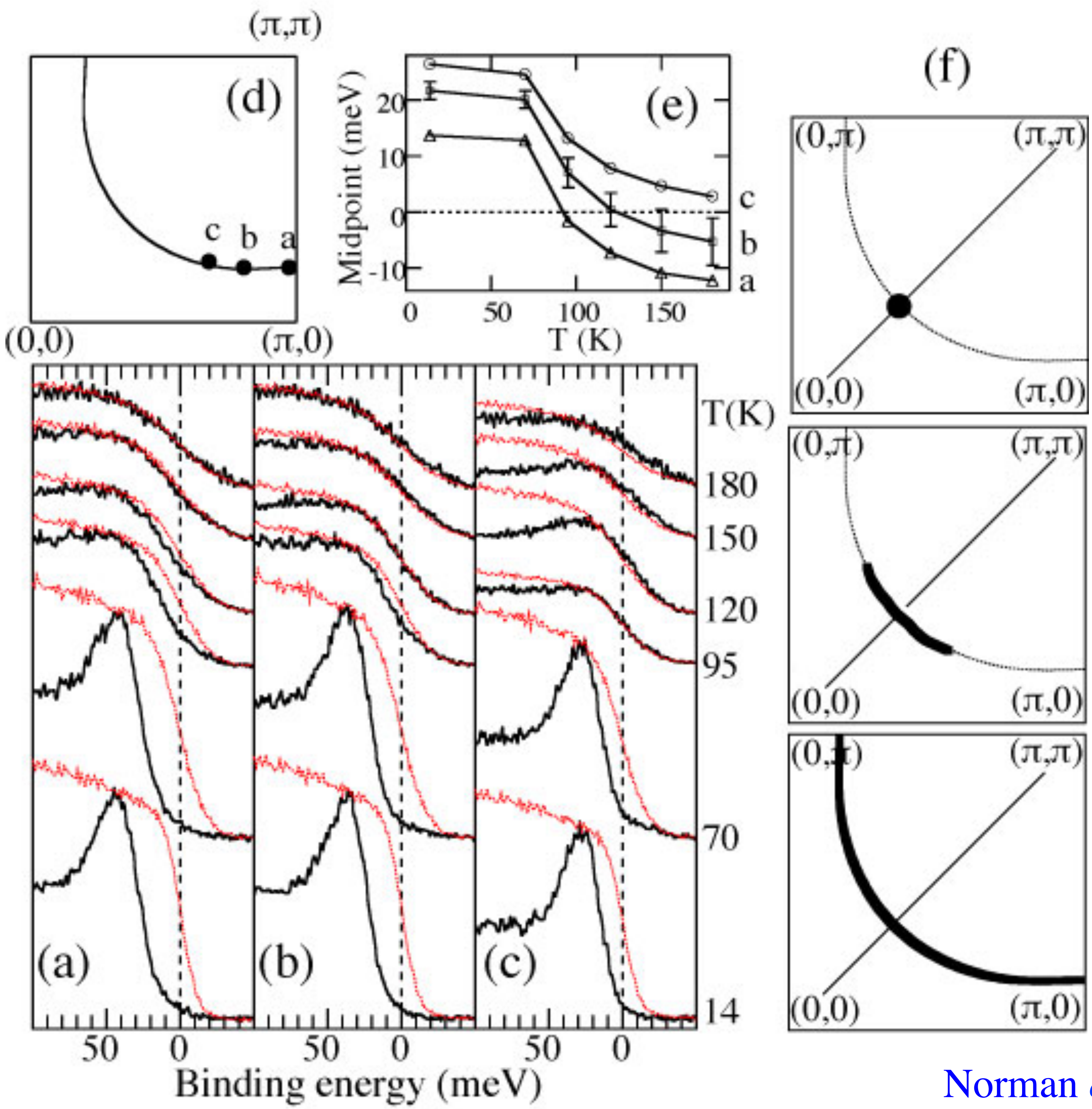
$$\boxed{\begin{aligned} G \sim & \begin{pmatrix} -v_F k_\perp + \omega + i\Gamma \cosh 2b_\infty & -\Delta + \Gamma \sinh 2b_\infty \\ -\Delta - \Gamma \sinh 2b_\infty & v_F k_\perp + \omega + i\Gamma \cosh 2b_\infty \end{pmatrix} \\ & \times \left[v_F^2 k_\perp^2 + \Gamma^2 + \Delta^2 - \omega^2 - 2i\Gamma\omega \cosh 2b_\infty \right]^{-1} \end{aligned}}$$



Vegh, arXiv:1007.0246

So, what have we learned?

? ? ?



$T < T_c$
(node)

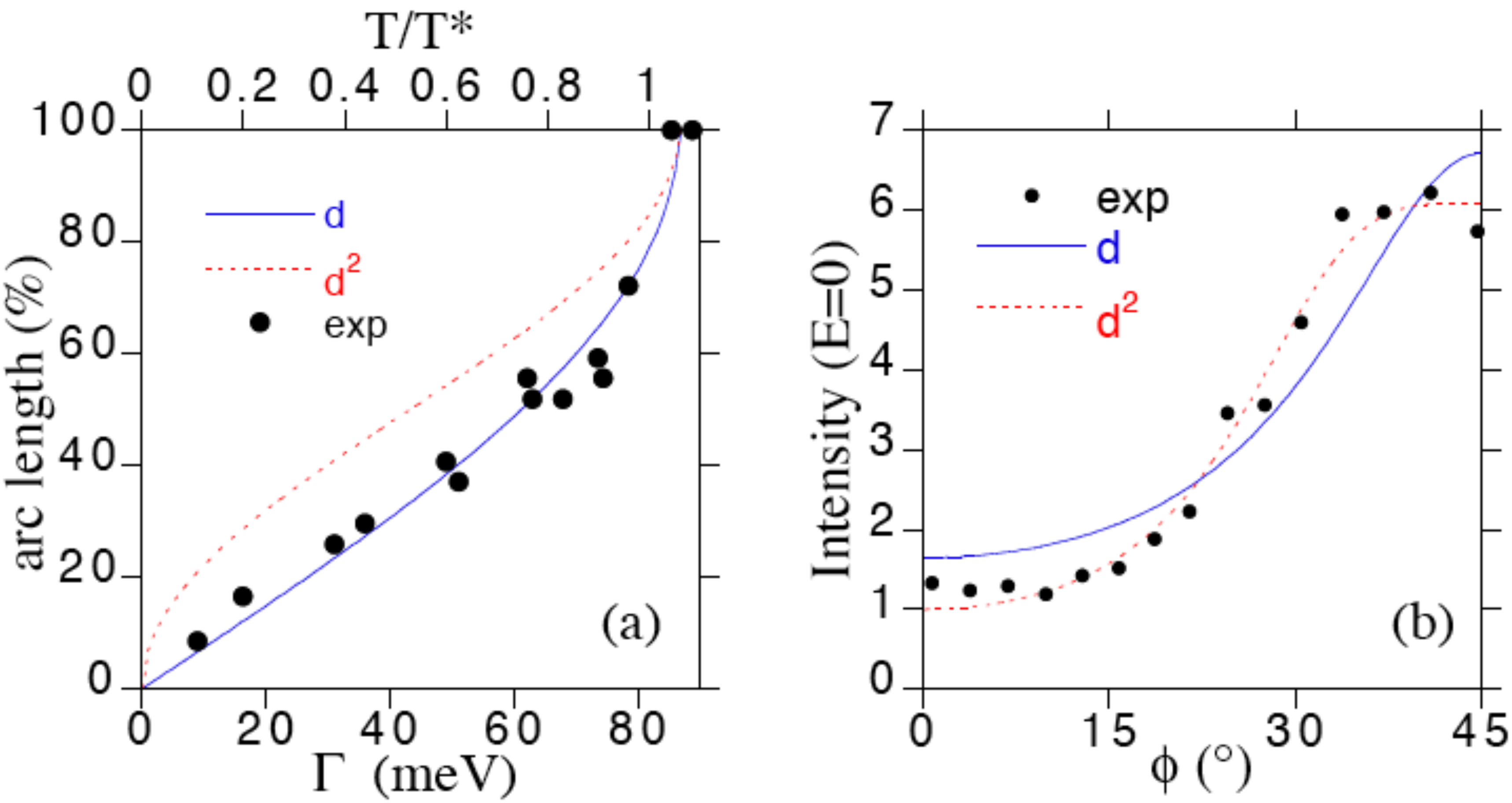
$T_c < T < T^*$
(Fermi arc)

$T > T^*$
(full Fermi surface)

Norman *et al.*, Nature (1998)

Arc Length is Linear in Γ -> $\Gamma \sim T$ -> Arc Length ~ T

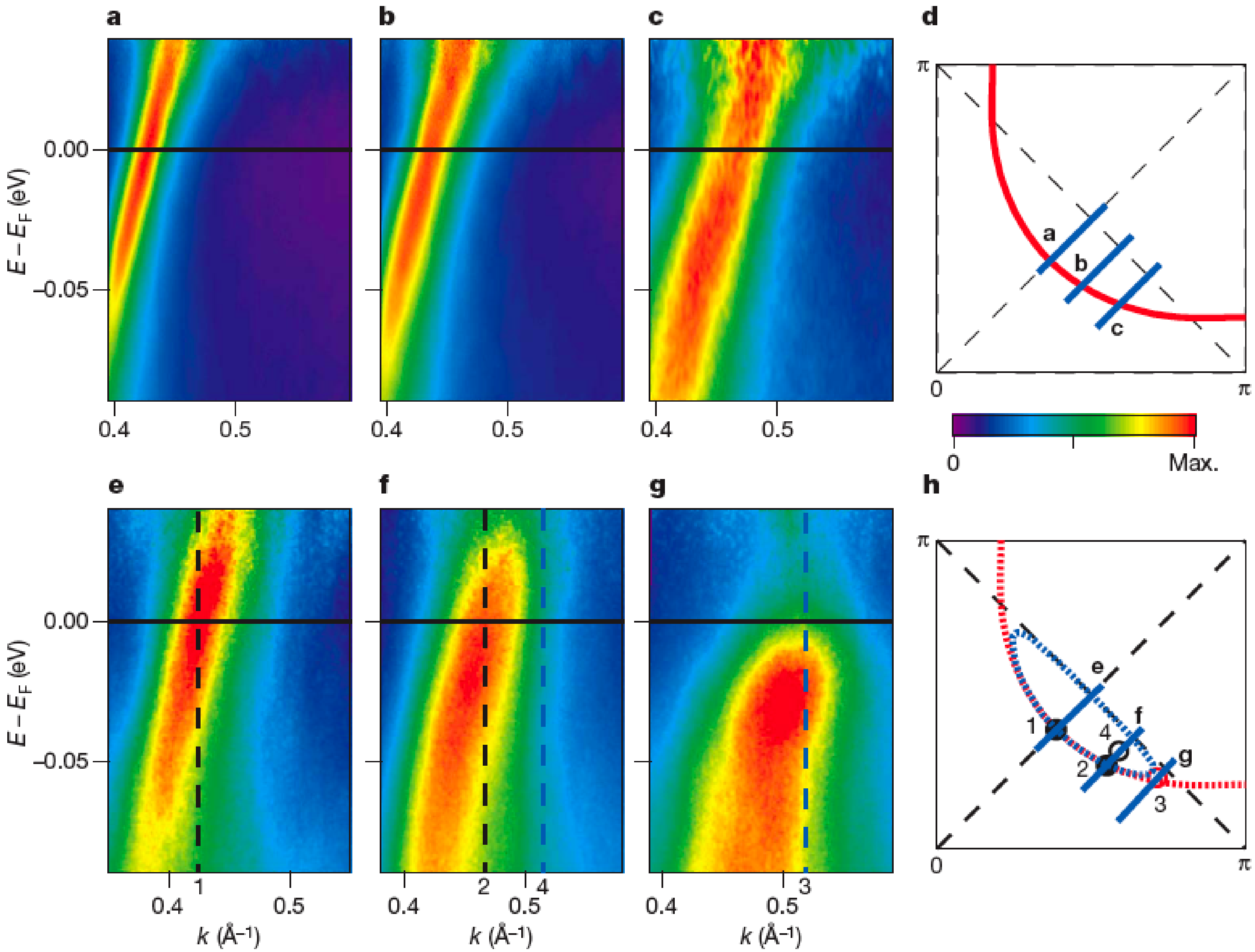
(lifetime broadened d-wave node)



Also explains arc collapse below T_c (Γ -> 0)

Norman *et al.*, PRB (2007)

Pocket in Bi2212?



Yang *et al*., Nature (2008)

The real question (at least in my mind) is arcs versus pockets

1. If arcs, what is the origin of the d-wave gap above T_c?

2. If pockets, what are their origin (stripes?)

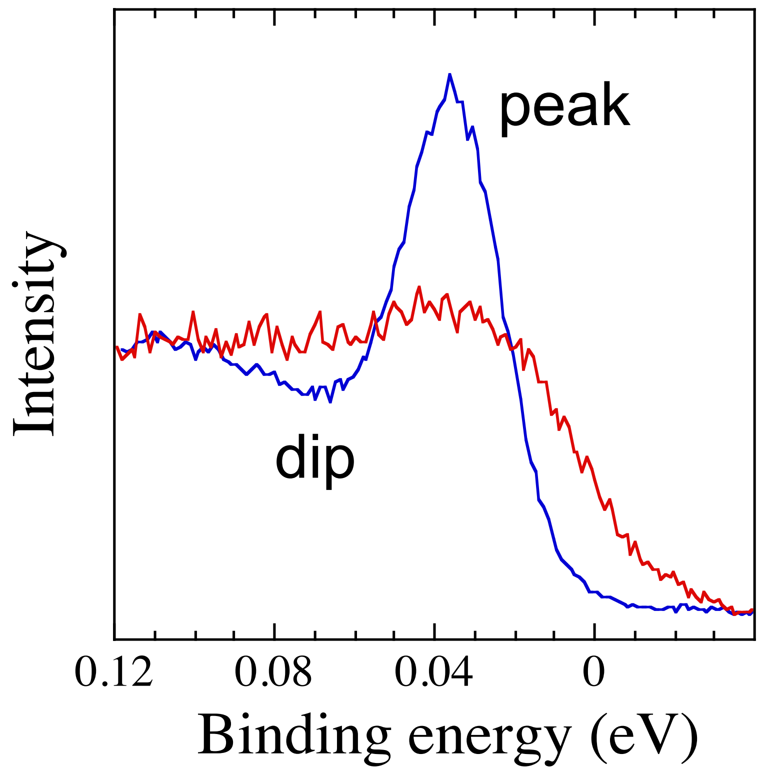
Contrast this with the AdS/CFT philosophy - just add an SDW field

$$\boxed{L = ig_\Phi \Phi^i \overline{\psi}_a (\sigma^i)_{ab} \psi_b .}$$

$$\boxed{S_1 = \int d^4x \sqrt{-g} \left[-\partial_M \Phi \partial^M \Phi - m_\Phi^2 \Phi^2 + i g_\Phi \Phi \overline{\zeta}_a (\sigma^z)_{ab} \zeta_b \right]}$$

Vegh, arXiv:1007.0246

Photoemission spectrum above and below T_c at momentum $k=(\pi,0)$ for Bi2212



Incoherent normal state

Coherent superconductor

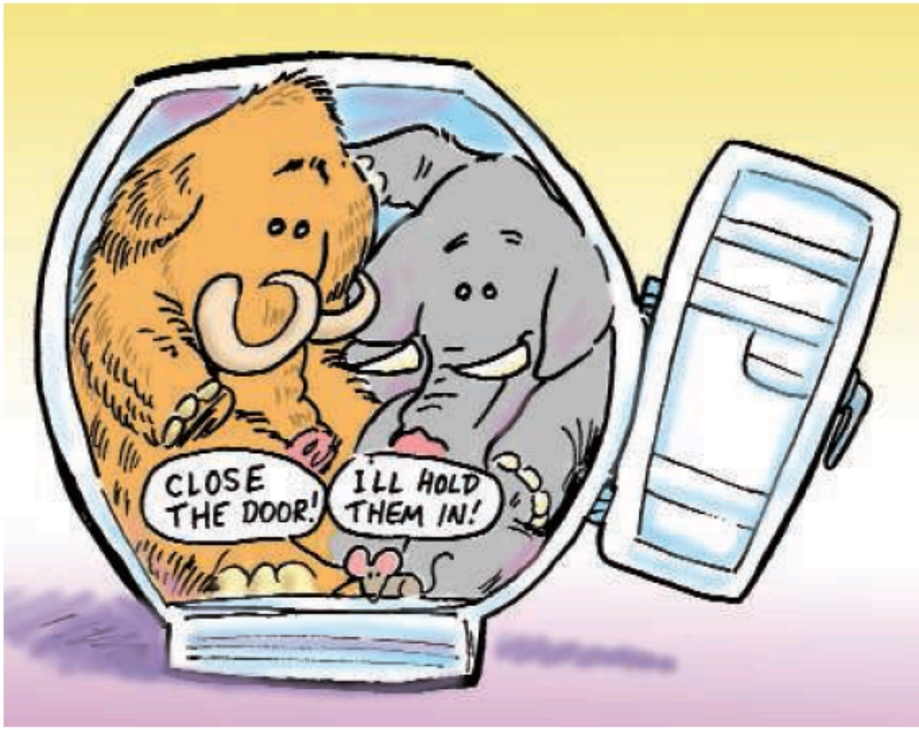
Norman *et al.*, PRL (1997)

What is the origin of the peak/dip/hump?

1. Bilayer splitting?
2. Scattering rate gap?
3. Coupling to spin fluctuations?
4. Coupling to current fluctuations?
5. Coupling to phonons?
6. Combination?

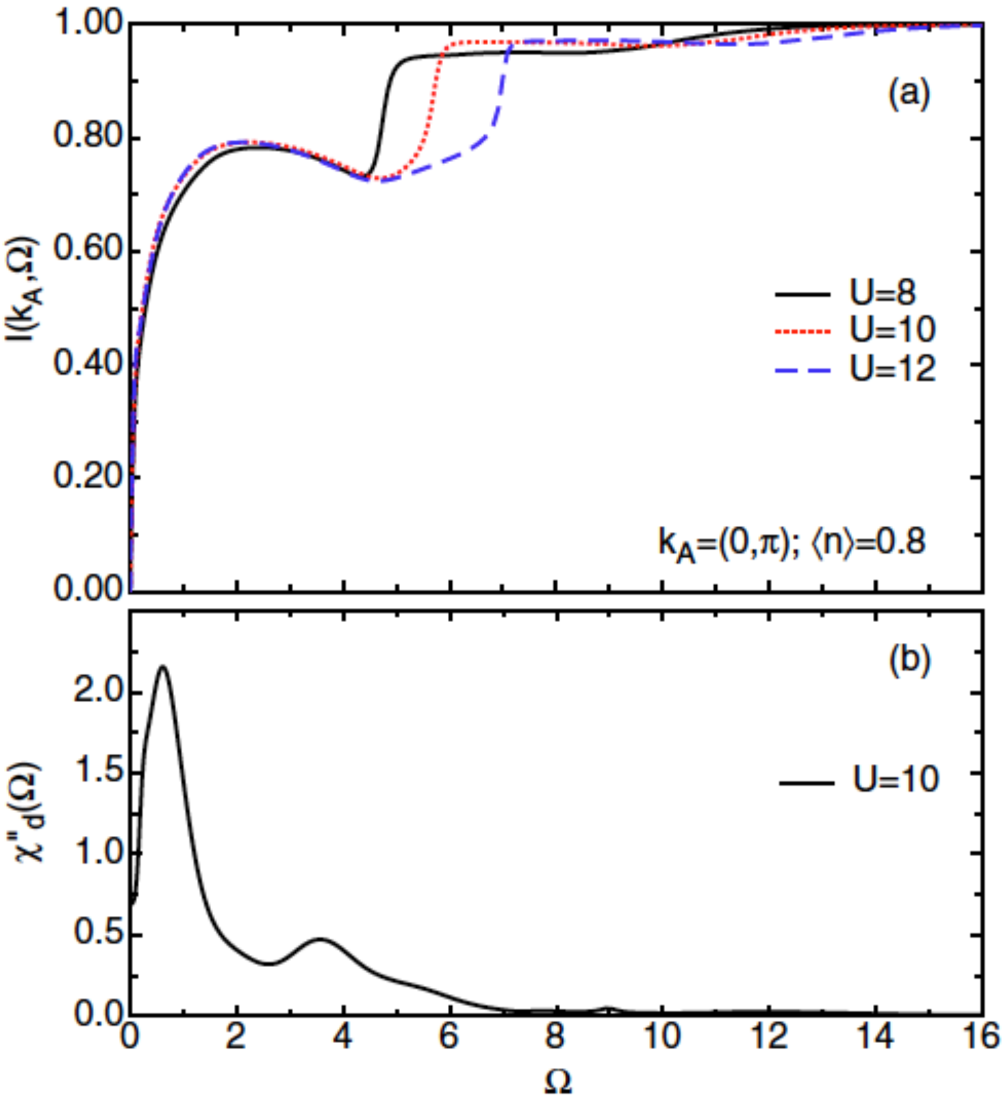
What is the origin of pairing?

Is there a pairing glue?



"We have a mammoth and an elephant in our refrigerator—
do we care much if there is also a mouse?"

Anderson, Science 317, 1705 (2007)



$$I(k_A, \Omega) = \frac{\frac{2}{\pi} \int_0^{\Omega} \frac{\phi_2(k_A, \omega')}{\omega'} d\omega'}{\frac{2}{\pi} \int_0^{\infty} \frac{\phi_2(k_A, \omega')}{\omega'} d\omega'}$$

Maier, Poilblanc, Scalapino, PRL 100, 237001 (2008)

Can AdS/CFT help us with "our" gauge theories?

(Read/Newns/Coleman for Kondo, RVB for cuprates, etc.)

1. AdS/CFT large N limit different from "our" large N limit

2. "Our" gauge fields are constraint fields!

   In particular, they do not have a "kinetic" energy (that is, one is at "infinite" coupling)

3. Can we find an AdS dual to such theories?

Nayak, PRL 85, 178 (2000)
Lee, Nagaosa, Wen, RMP 78, 17 (2006)